Comparison of Three Residential Regression Models: Additive, Multiplicative, and Nonlinear

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Abstract

This report describes the results of three citywide multiple regression models developed for single-family homes in the City of Calgary: an additive, a multiplicative, and a nonlinear (hybrid) model. The purpose of the project was to compare results from the three approaches to help determine which would provide better accuracy and uniformity in the city's future valuation efforts. All three methods produced good results and demonstrated certain advantages. The paper compares and evaluates the relative merits of each approach.

Database, Sales Edits, and Method

The city provided a database for the project that contained 40,805 sales from July 1999 through August 2001 (twenty-six months). Unfortunately the sales had not been edited to remove or identify invalid transfers, and sales prices ranged from zero to $10 billion. The sales were screened electronically in a multistage process to remove nonmarket and invalid transfers to the extent possible. The following sales were removed for purposes of the project:

- Sales below $10,000 or equal to $10 billion (1,137 sales)
- Duplicate transactions, for which the sale date, price, and all other data were identical (3,467 sales)
- Transfers involving nonresidential properties not modeled by the city (159 sales)
- Properties built in 2001 that may not have been complete at time of sale (28 sales)
- Properties for which the construction year equaled or exceeded the sale year and for which the price indicated that the sale was likely for land only
- Properties with sale codes suggesting that the sale may not have been representative of market value (for example, family sales and sales involving a financial institution)
- Transactions for which the current assessment–sale price ratio was below 0.50 or greater than 1.50 (4.0 percent of remaining sales)

After these sales were removed, the database contained 30,356 sales. An examination of graphs and descriptive statistics revealed several properties with implausible or extreme data, such as one square meter of basement area. Twelve such parcels were deleted. Finally, because of insufficient data to develop location adjustments, neighborhoods with less than 10 sales were excluded (there were 41 such sales). With these

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exclusions, the final database contained 30,303 usable sales for analysis. Although some valid transfers were inevitably removed during the electronic editing process and some non-arm's-length transfers undoubtedly still remained, the edited data provided a sound base for purposes of the project.

To provide a control group to compare results from the three modeling approaches objectively, the database was randomly split into a model and test group. (Without a control group, modeling methods that overly fit or chase sales tend to show up as artificially good. Location response techniques can be particularly prone to this direction.) The test group consisted of a random sample of 5,000 parcels of the 30,303 remaining sales, a sample large enough to evaluate thoroughly the results for various subgroups of property. The balance of sales (25,303) were retained in the model group and used to develop the models. Sales ratio statistics were calculated on both the model and test groups.

Each model was developed in a series of steps. First, a base model was developed using variables for lot size, living area, construction quality, age, neighborhood, and sale date. Second, a full exploratory model was developed using all available property characteristics except for subneighborhood codes, which often contained too few sales for analysis. In the additive model (first model developed), sales ratios were analyzed by subneighborhood codes and, where there were adequate sales and the ratios deviated significantly from 1.00, binary variables were created and added to the model. For consistency, these same variables were used in the multiplicative and nonlinear models as well. The final model was produced by purging the model of any variables with unreasonable coefficients or by combining and weighting variables for similar features. For example, attached, basement, and detached garage areas were combined into a single linearized garage variable in which basement and detached garage areas were set to a market-indicated percentage of attached garage area. When complete, the models were saved and applied to the holdout group, and sales ratio analyses were conducted on both the model and holdout samples.

Including neighborhood and subneighborhoods, the models contained several hundred variables and are too voluminous to present in their entirety. However, the paper presents the final nonlinear model, which is the model approach with which most practitioners are probably least familiar (all three models were based on the same property characteristics). The ensuing discussions focus on explaining the three modeling approaches and summarizing and contrasting sales ratio results achieved by each.

Additive Model

Additive models are easiest to calibrate and are the most common for single-family residential properties. In an additive model, the contribution of all components is added. Each component can employ transformations (such as raising a variable to a power or multiplying two variables together), but the contribution of each is *added*. Adjustments can be expressed on a per-unit basis (by multiplying a quality variable by a size variable), but percentage adjustments to land, building, or total values are not feasible.

A graph of sale price–assessment ratios with time indicated a linear pattern of modest inflation along with some seasonal variations. Therefore, a single variable, MONTHS (coded 1 to 26), was employed to capture time trends. Seasonality variables were created and tested for WINTER (November through February) and SPRGFALL (March, April, September, and October).

The base model included variables for quality/size (one size variable for each quality class), lot size, age, time, seasonality, and neighborhood (community) codes. Experimentation showed that raising both age and lot size to the power of 0.75 provided the best fit. All variables entered rationally except that average-plus quality entered higher than good quality (the variables were later combined). The model produced a median ratio of 1.007 and coefficient of dispersion (COD) of 8.60.

Next an exploratory model was developed using all candidate variables. Poor, average, and fair quality all entered at similar per-square-meter rates (about $535), and average-plus again entered higher than good quality. The other variables behaved pretty much as expected, with many premium location influences (view, waterfront, green belts, and so on) entering particularly strongly. The model indicated a time trend over the twenty-six-month period of 0.34 percent per month (coefficient for MONTHS of $640 divided by the average sale price of $189,086), or 1.0 percent per quarter. The model also indicated seasonality adjustments of –1.0 percent for winter months and –0.5 percent for spring and fall months. All sales were then adjusted to the assessment date of July 1, 2001, at these rates (July and August 2001 sales were adjusted slightly downward). Across all sales, the combination of time and seasonality produced an average adjustment of 4.46 percent (or average time-adjustment factor of 1.0446). For consistency, these same time adjustments were used in all subsequent models.

Using time-adjusted sales prices, the exploratory model was then rerun. As already mentioned, average-plus and good quality were combined, and garages were linearized into a single variable (LINGAR). The model produced a median ratio of 1.002 and COD of 7.36 (versus 8.60 for the base model). Sales ratios were analyzed by subneighborhood codes and additional binary variables created as appropriate. Some subneighborhood codes with few sales but poor ratios were combined with others exhibiting the same pattern. Without the benefit of first-hand knowledge of the market, this process was necessarily subjective and admittedly created a bias toward better performance (lower CODs) for the model group than could be sustained for the test group. Final sales ratio results for the additive model are shown in table 1.

Note that although the results are excellent, the COD for the holdout group (7.10) is not quite as good as for the model group (6.75), probably because of the tendency to create some subneighborhood variables based on few sales and no supporting appraisal knowledge. This illustrates the potential pitfalls in general of basing mass appraisal adjustments on too few sales. In any case, for consistency, the same subneighborhood variables were employed in the multiplicative and nonlinear models as well.

Figures 1–6 show plots of the ratios for the 5,000 sales in the test group against key variables: living area, lot size, age, construction quality, building style, and value (defined as one-half of time-adjusted sale price plus one-half of model-estimated value). The graphs indicate strong equity across key variables. Similar graphs were run on the holdout group for the multiplicative and nonlinear models with similarly good results.

Table 1
Sales Ratio Results: Additive Model

Ratio Test	Number	Median	Mean	Minimum	Maximum	Standard deviation	COD
.00	25303	1.0022	1.0066	.50	2.73	.093	6.75
1.00	5000	1.0026	1.0057	.49	1.92	.098	7.10
Total	30303	1.0023	1.0064	.49	2.73	.094	6.81

Figure 1
Graph of Ratios with Size
Final Additive Model—Holdout Group

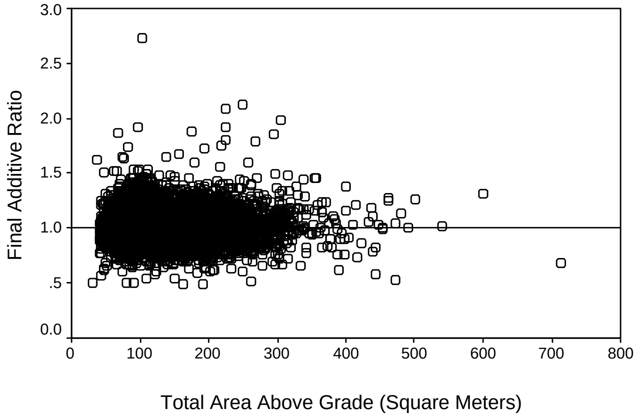


Figure 2
Graph of Ratios with Age
Final Additive Model—Holdout Group

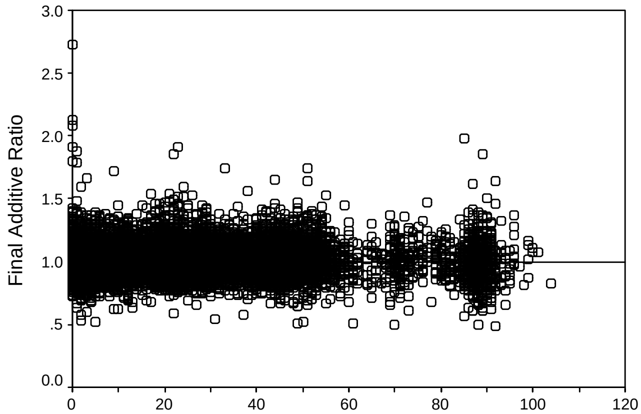


Figure 3
Plot of Ratios with Quality—Holdout Group

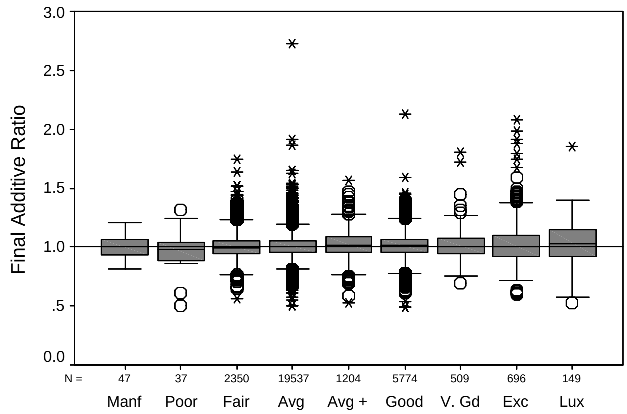


Figure 4
Plot of Ratios with Style—Holdout Group

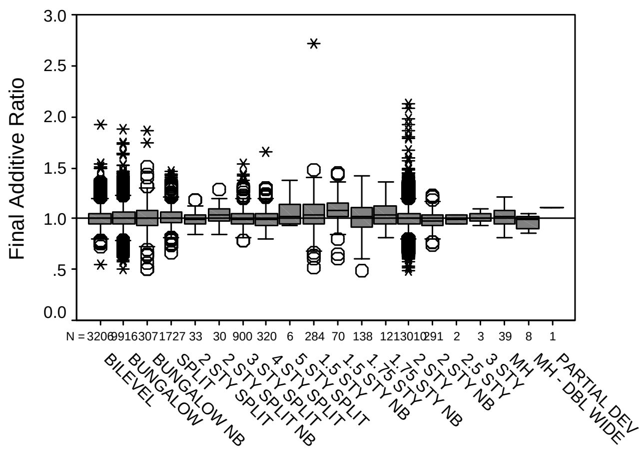


Figure 5
Graph of Ratios with Lot Size
Final Additive Model—Holdout Group

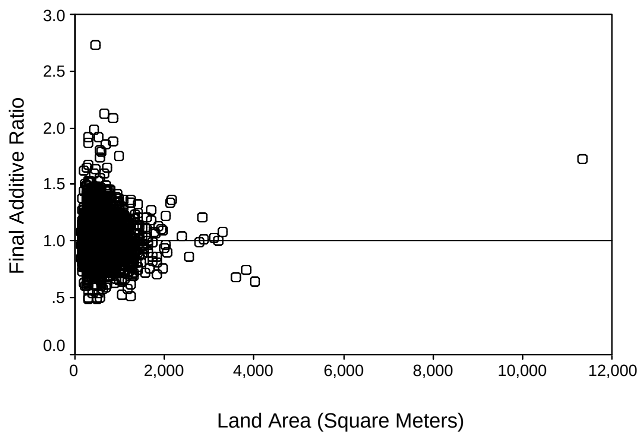
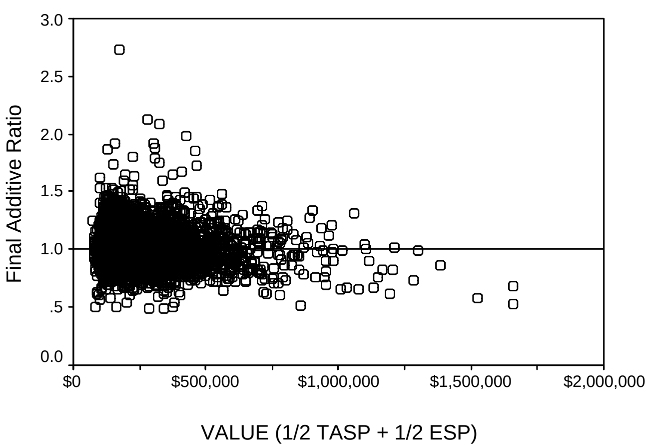


Figure 6
Graph of Ratios with Value
Final Additive Model—Holdout Group



Multiplicative Model

Multiplicative models have several advantages. They readily accommodate percentage adjustments and efficiently calibrate nonlinearities. Also, because the models are in logarithmic format, the range of the dependent variable is considerably reduced, meaning that more equal weight is given to each property and the influence of outliers is reduced. On the negative side, logarithms are involved, making the math more complex, and inherently additive relationships can be difficult to accommodate. In particular, land and building variables must be multiplied together.

In order to accommodate various size variables, which are not inherently additive, all secondary areas were expressed relative to main living area. For example, basement area was expressed as the multiplier: 1 + BSMTAREA/SIZETOTL. The model then calibrated the exponent for the variable, which would be expected to be greater than zero but less than one. In this case, the final model calibrated an exponent of .080, meaning that basement area is worth roughly 8 percent as much as main living area (binary variables for style and no basement were also significant in the model). Lot size was handled in a similar manner, with the land/building area ratio divided by 4 to center it on the typical ratio. The final model calibrated an exponent for this variable of .113, meaning, for example, that a property with a land/building area ratio of 8 (twice the typical) would have a multiplier of 1.0815 (2.0 raised to the .113 power).

Binary variables are readily accommodated in multiplicative models, requiring no additional transformations. The model calibrates their associated multipliers. For example, the final model calibrated multipliers of 1.285 for excellent construction quality, 0.777 for manufactured homes, 1.049 for a walkout basement, 1.137 for location next to a golf course, and 0.954 for location on a major street.

Age adjustments require conversion to a percent good factor in multiplicative and nonlinear models. Age was raised to the power of 0.75 (optimal transformation in the additive models), divided by 100, and subtracted from 1. For example, the *initial* percent good factor calculated for a property with a fifty-year-old building was 0.812 ($1 - 50^{.75}/100$). The final model calibrated an exponent of 0.765 for the variable, so that the *calibrated* multiplier for a fifty-year-old building is 0.853 ($0.812^{.765}$). Note that in a multiplicative model the depreciation adjustment is applied to the entire property (whereas in a nonlinear model it would be applied only to building value).

Like additive models, multiplicative models are calibrated with additive regression, since taking logarithms converts them to additive format. Predicted values can then be converted to ordinary numbers by taking their antilog or exponential.

The base multiplicative model produced a median ratio of 1.003 and COD of 8.15 (versus 1.007 and 8.60 for the base additive model). The final model employed the same neighborhood and subneighborhood variables used in the additive model. Interestingly, the exponent calibrated for building size was .530, indicating considerable economies of scale (partly due to the fact that an increase in building size does not imply an increase in land size). As in the additive model, various location variables for green space, traffic, and so forth were strong. The highest multiplier was 1.587 for lake frontage (based on 71 sales). Sales ratio results for the final multiplicative model are shown in table 2.

Perhaps surprisingly, the COD for the holdout group (6.61) is significantly better than for the additive model (7.10). This illustrates the advantage of percentage adjustments across a database comprising an entire major metropolitan area: dollar and dollar-per-unit adjustments tend to break down across a heterogeneous database. It also indicates that the ratio variables used to accommodate basement, garage, and other size variables worked effectively and that the model suffered little (if any) by treating land and building terms interactively. Plots of ratios against key property characteristics showed horizontal and vertical equity at least as good as that achieved by the additive model. In all, the multiplicative model achieved impressive results.

Nonlinear Model

Nonlinear, or hybrid, models combine the best features of additive and multiplicative models, allowing the model builder to specify both additive and multiplicative relationships. However, there are drawbacks. First, software is comparatively limited. Second, hybrid models are more difficult to calibrate. Calibration requires a processor-intensive trial-and-error process. Also, the models do not contain the full range of features and diagnostics available with standard multiple regression analysis (MRA). In particular, stepwise options are not available. Fortunately, SPSS contains a nonlinear MRA module, which was used to specify and calibrate the models reported here.

The nonlinear models were formulated so as to be decomposable into land and building values. The land portion begins with a base-lot value, which represents the typical value of a typical lot (475 square meters) in a base neighborhood. In the final model, this estimate was $78,100 (Canadian dollars), which was then adjusted for size, corner location, topography, view, lake frontage, green space, traffic, and other influences. The building portion of the model contained both additive and multiplicative components. Contributions of the various size-related variables (main living area, basement area, garage area, and so on) were added and then adjusted for various quality features. Adjustments for building style and a building economy-of-scale factor were applied against main living area only, whereas property type, quality class, and depreciation adjustments were applied against the entire building. Neighborhood factors were applied against both land and buildings.

The final hybrid model is shown in the appendix. Note that SPSS reports the coefficient for each variable, along with its standard error and an approximate 95 percent confidence interval. As mentioned, the base land value was $78,100. Adjustments for location influences were considerably higher than they were in the multiplicative model, where they were applied to both land and building values. The base rate for building area is $839.90, and the building size factor is –.208, which indicates modest economy-of-scale adjustments: homes smaller than the base (120 square meters) would receive an upward per-square-meter adjustment, and larger homes would receive a downward adjustment. The exponent for the percent-good factor is 1.206, indicating that the initial percent-good factors (described earlier) would be somewhat magnified, for example, a fifty-year-old building, with an initial factor of .812 ($1 - 50^{.75}/100$) would assume a calibrated factor of 0.778 ($.812^{1.206}$). Sales ratio results for the nonlinear model are shown in table 3.

Table 2
Sales Ratio Results: Multiplicative Model

Ratio Test	Number	Median	Mean	Minimum	Maximum	Standard deviation	Sum
.00	25303	1.0018	1.0037	.49	2.55	.087	6.27
1.00	5000	1.0011	1.0030	.48	2.01	.092	6.61
Total	30303	1.0017	1.0036	.48	2.55	.087	6.33

Table 3
Sales Ratio Results: Nonlinear Model

Ratio Test	Number	Median	Mean	Minimum	Maximum	Standard deviation	COD
.00	25303	1.0022	1.0069	.49	2.63	.0914	6.67
1.00	5000	1.0026	1.0066	.49	2.02	.0963	6.97
Total	30303	1.0023	1.0068	.49	2.63	.0922	6.71

Table 4
Coefficients of Dispersion

	Model file	Test file
Additive model	6.75	7.10
Multiplicative model	6.27	6.61
Hybrid model	6.67	6.97

The CODs are better than in the additive model, but about a quarter point worse than those achieved by the multiplicative model. Again, horizontal and vertical equity were excellent.

Conclusions and Recommendations

All three models consistently produced median ratios near 1.000 for both the model and test data sets. CODs are shown in table 4. On the basis of these results alone, the multiplicative approach emerges as best. As discussed previously, in part this is because of the heterogeneous nature of the database. Interestingly, the results of the research thus indirectly suggest that one model is not likely to fit the entire city satisfactorily. Time-adjusted sales prices in the various communities ranged from $127,880 to $812,278. One set of additive adjustments cannot be expected to fit so broad a market.

Although the multiplicative model performed best, multiplicative models have potential explanatory problems for residential properties, because all variables are multiplied together. According to appraisal theory, some variables (those that are size-related or quantitative in nature) should be added, but others (those that are qualitative in nature) are best multiplied. Hybrid, or nonlinear, models are the most flexible and adaptable to this theory (additive models can approximate percentage adjustments through per-unit adjustments). Finally, multiplicative models require logarithms. Although this is not a serious complication, it is probably best to avoid the additional mathematics if there is no significant gain in model accuracy (as may be the case when market area models are developed).

Thus, although the multiplicative models clearly performed best on a global basis, we cannot conclude that it is the best approach for future models, either for the City of Calgary or elsewhere. Clearly, if a single model is to be developed, statistically at least, a multiplicative approach seems best. However, regional or market area models can better fit the various market segments that compose a metropolitan area. In this scenario, additive and hybrid models can be expected to fare comparatively better than at the global level. In the end, one must make a reasonable choice of a preferred modeling strategy based on test results, available resources and skills, and personal preferences. Happily, all three approaches can produce excellent (and generally similar) values for residential properties.

Appendix
Final Nonlinear Model

Run stopped after eight model evaluations and four derivative evaluations.
Iterations have been stopped because the relative reduction between successive residual sums of squares is at most SSCON = .00010000.

Source	Degrees of freedom	Sum of squares (SS)	Mean square
Regression	308	1.138034E+15	3694916612713
Residual	24995	1.386057E+13	554533897.735
Uncorrected total	25303	1.151895E+15	
(Corrected total)	25302	1.655654E+14	

R^2 = 1 - Residual SS/Corrected SS = .91628

Parameter	Estimate	Asymptotic standard error	Asymptotic 95 % confidence interval Lower	Upper
BLV	78100.754563	2211.2710996	73766.534338	82434.974787
LSIZ_EXP	.361883578	.010967300	.340387031	.383380125
CRL_FAC	.950287747	.005854308	.938812961	.961762533
SPF_FAC	.941477822	.021429870	.899474028	.983481616
TOPO_FAC	.923100268	.026164898	.871815543	.974384992
VIEW2FAC	1.751014591	.052452689	1.648204265	1.853824918
VIEW3FAC	1.576937256	.031606764	1.514986158	1.638888355
VIEW4FAC	1.417139739	.043172756	1.332518620	1.501760858
VIEW5FAC	1.156846748	.030577522	1.096913022	1.216780473
VIEWOFAC	1.332099925	.016370308	1.300013168	1.364186683
GOLF_FAC	1.292275157	.020131617	1.252816014	1.331734300
LAKE_FAC	3.230518599	.067119088	3.098961276	3.362075923
LAKEACSS	1.861119591	.051154199	1.760854380	1.961384802
GREENSPC	1.102345446	.008612199	1.085465034	1.119225859
GRP_FAC	1.237667815	.019735596	1.198984897	1.276350732
GRS_FAC	1.101775330	.014012558	1.074309900	1.129240759
COMM_FAC	.937123297	.012921572	.911796262	.962450332
MFY_FAC	.914875718	.011677154	.891987816	.937763621
TRANSTWR	.787504623	.029597055	.729492670	.845516577
TRM_FAC	.872633665	.008929493	.855131338	.890135992
TRC_FAC	.954805414	.004998891	.945007297	.964603531
TRE_FAC	.877176401	.011672257	.854298097	.900054705
B1	839.90537908	19.833097476	801.03135219	878.77940598
BSIZ_EXP	–.207896369	.016857242	–.240937546	–.174855192
BI_LEVEL	.993874612	.006705053	.980732318	1.007016906
SPLIT2	.927200708	.026976321	.874325547	.980075869
SPLIT45	.907889942	.013576740	.881278740	.934501144
STY_15	1.062649326	.011285665	1.040528765	1.084769887
STY_175	1.141031643	.018998673	1.103793138	1.178270149
STY_2PLS	.878496038	.004799533	.869088673	.887903403
NOBSMT	.898845235	.011247259	.876799952	.920890517
WALKOUT	1.100188722	.004791795	1.090796524	1.109580920
BASEMENT	131.51164807	5.429305754	120.86989238	142.15340375
VERAND	290.36138345	32.543105212	226.57500067	354.14776622
GARAGE	219.49395437	11.331167102	197.28420649	241.70370225
CARPORT	14.150765129	46.985323100	–77.94320637	106.24473663
DECK	147.58486573	14.145512611	119.85883663	175.31089482
FIREPLAC	3252.1762335	345.95389303	2574.0864416	3930.2660255
Q_POOR	.788478585	.084358006	.623131977	.953825193
Q_FAIR	.970508806	.009894263	.951115474	.989902137
Q_AVEPGD	1.093029628	.004988800	1.083251288	1.102807967
Q_GDPLS	1.249600936	.009966629	1.230065762	1.269136110
Q_EXC	1.378140722	.010438950	1.357679771	1.398601673
Q_LUX	1.746363215	.017176332	1.712696604	1.780029825
MANF_HM	.567608706	.054498524	.460788423	.674428989
CONDOFAC	1.296596953	.025707760	1.246208246	1.346985660
SEMI_FAC	.938309048	.006731960	.925114014	.951504082
ROWH_FAC	.928055884	.043582555	.842631537	1.013480231
CDU_FAPR	.799900184	.015198907	.770109441	.829690927
CDU_GOOD	1.097394217	.009615777	1.078546733	1.116241701
PGD_EXP	1.205636513	.037893913	1.131362235	1.279910791
N_ABB3	.951847884	.056385767	.841328494	1.062367274
N_ABBX	.892845754	.011001200	.871282762	.914408747
N_ACA134	1.102850114	.038446205	1.027493312	1.178206916
N_ACAX	1.035790519	.011327228	1.013588492	1.057992546
N_ALBX	.902723092	.014757362	.873797803	.931648381

Parameter	Estimate	Asymptotic standard error	Asymptotic 95 % confidence interval Lower	Upper
N_ALT1	1.598596990	.045287999	1.509829872	1.687364107
N_ALTX	1.296189376	.008856421	1.278830274	1.313548477
N_APP23	.984678198	.020886775	.943738902	1.025617495
N_APPX	.970479308	.010684481	.949537104	.991421513
N_ARB12	1.100022468	.015122723	1.070381050	1.129663887
N_ARB345	1.085099342	.016193044	1.053360033	1.116838651
N_ARB9	1.039546174	.015788748	1.008599308	1.070493039
N_BED368	.916547223	.032392896	.853055258	.980039187
N_BED9X	.968957618	.015605820	.938369301	.999545935
N_BEDX	.944954237	.008701602	.927898590	.962009885
N_BEL23	1.922211838	.024358669	1.874467427	1.969956248
N_BELX	1.654493656	.023733467	1.607974677	1.701012635
N_BNF457	1.199146738	.033061798	1.134343687	1.263949788
N_BNFX	1.332683064	.018459246	1.296501867	1.368864261
N_BNK1	1.282804473	.075757834	1.134314703	1.431294244
N_BNKX	1.430010077	.018161103	1.394413257	1.465606896
COM_BOW	1.043357039	.009847289	1.024055778	1.062658300
N_BRD4	1.218931365	.058615910	1.104040765	1.333821966
N_BRDX	1.262186024	.018916587	1.225108412	1.299263636
N_BRE2	1.139556137	.025860789	1.088867483	1.190244791
N_BREX	1.197429770	.012930864	1.172084524	1.222775016
N_BRT2	1.511442562	.090904211	1.333265010	1.689620114
N_BRTX	2.091839272	.020814201	2.051042225	2.132636319
COM_BYV	1.514999381	.020924442	1.473986254	1.556012507
COM_CAM	1.216302558	.022225806	1.172738684	1.259866433
N_CAN34	1.036953574	.011727926	1.013966155	1.059940993
N_CAP456	1.253706761	.034233536	1.186607036	1.320806486
N_CAPX	1.333530932	.017892981	1.298459647	1.368602217
N_CAS36	.892869507	.014918992	.863627413	.922111600
N_CAS8	.866310731	.025338328	.816646132	.915975330
N_CASX	.927229361	.013498767	.900770992	.953687731
N_CHA6	.977304415	.034184591	.910300624	1.044308205
N_CHA10X	1.148976116	.021188892	1.107444653	1.190507579
N_CHA14	.892861585	.046589560	.801543333	.984179837
N_CHAX	1.010281022	.006145653	.998235185	1.022326860
N_CHKX	1.367239054	.020298522	1.327452769	1.407025339
N_CHR2	1.122112076	.026587661	1.069998711	1.174225440
N_CHRX	1.274517172	.012212009	1.250580922	1.298453423
COM_CHW	1.193184350	.014712654	1.164346692	1.222022009
N_CIT34	1.050467700	.017694563	1.015785324	1.085150075
COM_CLI	1.723774361	.038299275	1.648705550	1.798843172
N_COA1	1.026849496	.020077068	.987497273	1.066201718
N_COAX	1.066415932	.015490513	1.036053623	1.096778242
N_COL2	1.534888829	.040318490	1.455862239	1.613915420
N_COLX	1.192136330	.018687912	1.155506933	1.228765727
COM_CON	1.317809675	.062749741	1.194816526	1.440802824
N_COR12	.805198373	.016368776	.773114618	.837282128
N_COU3	.953644149	.012066726	.929992663	.977295635
N_COV19	.955554890	.014307133	.927512077	.983597704
N_CRE10X	1.788951616	.055314246	1.680532470	1.897370762
N_CREX	1.376901880	.017008189	1.343564839	1.410238920
COM_DAL	1.079298659	.010849681	1.058032653	1.100564665
COM_DIA	1.063452019	.017319214	1.029505349	1.097398688
COM_DOU	1.033617429	.005727607	1.022390986	1.044843872
N_DOV2	.880562020	.059331718	.764268395	.996855644
N_DOVX	.872493062	.009595516	.853685291	.891300832

Parameter	Estimate	Asymptotic standard error	Asymptotic 95 % confidence interval Lower	Upper
N_DRG34X	.908760847	.030395330	.849184228	.968337465
N_DRGX	.960735572	.014421801	.932468002	.989003143
N_DRN35	1.071626793	.035996130	1.001072280	1.142181307
N_DRNX	.957291100	.012509019	.932772695	.981809506
N_EDGX	1.128572779	.006317342	1.116190421	1.140955137
N_EPK245	2.184458200	.019596395	2.146048124	2.222868276
N_EPK37	1.647722106	.028177669	1.592492232	1.702951980
N_EPKX	2.017860301	.015670879	1.987144465	2.048576138
N_ERI367	.935885252	.010609879	.915089271	.956681232
N_ERIX	.934091920	.018826564	.897190757	.970993083
N_ERL3	1.976659519	.057572411	1.863814237	2.089504800
N_ERLX	1.333989293	.050127067	1.235737321	1.432241266
N_EVE14X	1.243107024	.016444152	1.210875527	1.275338521
N_EVEX	1.084581416	.008693887	1.067540891	1.101621941
N_EYA34	2.023817678	.043417064	1.938717702	2.108917654
N_EYA8	1.156337950	.106605941	.947384092	1.365291808
N_EYAX	1.499859856	.020704797	1.459277248	1.540442464
N_FAI2	1.103928477	.064295847	.977904869	1.229952085
N_FAIX	1.030594324	.015411198	1.000387477	1.060801170
N_FAL2	.928563374	.017214640	.894821677	.962305071
N_FALX	.875116010	.010249273	.855026838	.895205181
N_FHT56X	.851153442	.021523398	.808966327	.893340558
N_FHT8X	.932983771	.036861961	.860732179	1.005235363
N_FHTX	.876090452	.020057707	.836776178	.915404726
N_FLNX	.893485025	.011584311	.870779100	.916190949
COM_GBK	1.137098118	.015106461	1.107488574	1.166707662
N_GDL5	1.468784362	.063757814	1.343815331	1.593753392
N_GDLX	1.164183955	.017360759	1.130155856	1.198212054
N_GLA67	1.113754938	.038304482	1.038675921	1.188833955
N_GLAX	1.150982690	.015793151	1.120027193	1.181938187
COM_GRV	1.064536091	.038497556	.989078638	1.139993544
COM_HAM	1.125162519	.007674938	1.110119193	1.140205844
N_HAR2	.654771071	.030013613	.595942641	.713599501
N_HAR467	.931574108	.011258307	.909507170	.953641047
N_HAW123	1.068195479	.015901143	1.037028313	1.099362646
N_HAYX	1.093355279	.012359551	1.069129839	1.117580719
N_HID3	1.025544559	.033988595	.958924933	1.092164186
N_HID4	.656877894	.028403778	.601204833	.712550954
N_HIL1	1.568759195	.029767828	1.510412518	1.627105872
N_HIL6X	1.466158973	.041571927	1.384675573	1.547642373
N_HILX	1.528515781	.016541395	1.496093683	1.560937880
N_HIW2	1.107366905	.029284271	1.049968027	1.164765782
N_HIWX	1.162587734	.022475582	1.118534284	1.206641185
N_HOU26	1.415921439	.049195989	1.319494435	1.512348444
N_HOU35	1.209419353	.056847422	1.097995093	1.320843612
N_HOU8X	1.481437588	.028700476	1.425182982	1.537692194
N_HOUX	1.297787844	.027429985	1.244023475	1.351552213
N_HPKX	1.135015628	.018441714	1.098868794	1.171162461
N_HUN89X	.935727597	.024946885	.886830249	.984624945
N_HUNX	.981395476	.009405791	.962959577	.999831375
N_ING5	1.184953190	.030799161	1.124585039	1.245321341
N_INGX	1.411120437	.021381483	1.369211484	1.453029391
N_KEL3	1.381891253	.050079479	1.283732555	1.480049951
N_KELX	1.441487069	.021026329	1.400274238	1.482699899
COM_KIL	1.242328122	.011424008	1.219936400	1.264719844
COM_KIN	1.052401563	.017323744	1.018446015	1.086357111

Parameter	Estimate	Asymptotic standard error	Asymptotic 95 % confidence interval Lower	Upper
N_LKB23X	1.302317083	.010356751	1.282017247	1.322616918
N_LKBX	1.169549815	.007924497	1.154017340	1.185082291
N_LKV379	1.774987982	.020367280	1.735066927	1.814909037
N_LKVX	1.201796289	.013569949	1.175198399	1.228394179
N_MAF345	1.393926678	.030075306	1.334977326	1.452876030
N_MAFX	1.778030491	.023982207	1.731023968	1.825037014
N_MAL311	1.054497082	.025777813	1.003971066	1.105023098
N_MCK78X	1.280778404	.017358253	1.246755217	1.314801592
N_MCKX	1.078126289	.005815473	1.066727622	1.089524956
N_MCTX	1.028302758	.014157924	1.000552401	1.056053115
N_MEA23	1.127486079	.052744726	1.024103343	1.230868815
N_MEAX	1.375651412	.032387420	1.312170181	1.439132643
N_MID14X	1.091229866	.033803649	1.024972744	1.157486989
N_MIDX	.968340808	.008770937	.951149260	.985532357
COM_MIS	1.834364349	.044395194	1.747347181	1.921381517
N_MLR246	.967908875	.012291375	.943817064	.992000686
N_MLRX	.972611093	.011928669	.949230206	.995991980
COM_MON	1.056089133	.013581924	1.029467771	1.082710495
N_MOP2	1.223324300	.040558680	1.143826923	1.302821677
N_MOPX	1.299062942	.013449523	1.272701093	1.325424790
N_MOR268	1.088216224	.040797275	1.008251187	1.168181261
N_MPKX	.911808055	.009812303	.892575370	.931040740
N_MPL45	1.087155067	.041724228	1.005373149	1.168936986
N_MPLX	1.101488440	.015974410	1.070177666	1.132799215
COM_MRL	.914973828	.010017732	.895338490	.934609166
N_MRT23	.968961944	.014858157	.939839090	.998084798
N_MRTX	.942243848	.007853260	.926851000	.957636696
N_NGM12X	1.320767552	.044672169	1.233207499	1.408327606
N_NGMX	1.158665757	.019601796	1.120245094	1.197086420
COM_NHU	1.117309188	.031251406	1.056054612	1.178563765
N_NHVX	1.111044221	.020074456	1.071697117	1.150391324
N_OAK2	1.231721947	.026799702	1.179192969	1.284250925
N_OAK3	1.066893088	.014983332	1.037524885	1.096261291
N_OAKX	1.139464104	.013894789	1.112229508	1.166698700
N_OGD35X	.885719944	.041903768	.803586117	.967853772
N_OGD7	.994276166	.057038982	.882476437	1.106075896
N_OGDX	.943764741	.010311736	.923553138	.963976344
COM_PAL	1.120505917	.023770999	1.073913375	1.167098459
N_PAN1	.974261395	.009086393	.956451535	.992071255
N_PAT1X	1.190613817	.012956149	1.165219009	1.216008625
N_PAT20	1.170124498	.046983369	1.078034357	1.262214638
N_PATX	1.110079988	.011671793	1.087202594	1.132957381
N_PEN156	.914380316	.045131113	.825920703	1.002839928
N_PEN38X	.818125951	.031742764	.755908284	.880343617
N_PENX	.873863066	.010699762	.852890908	.894835224
N_PIN3	.929583710	.038691444	.853746225	1.005421194
N_PINX	.892188475	.009479132	.873608825	.910768126
N_PKD135	1.274915386	.042442090	1.191726416	1.358104355
N_PKDX	1.404690471	.027195788	1.351385141	1.457995801
N_PKH16X	1.750802410	.020139595	1.711327630	1.790277191
N_PKH3	2.257633211	.049680488	2.160256560	2.355009862
N_PKHX	1.317090281	.044175426	1.230503871	1.403676692
COM_PKL	1.083966823	.010131210	1.064109060	1.103824585
COM_PUM	1.287506189	.012960359	1.262103130	1.312909248
N_QLD26	.961220309	.021833156	.918426051	1.004014567
N_QLDX	.917295920	.015096293	.887706306	.946885535
N_RAM36	1.058161804	.054549931	.951240760	1.165082848

Parameter	Estimate	Asymptotic standard error	Asymptotic 95 % confidence interval Lower	Upper
N_RAMX	1.286311236	.017907910	1.251210688	1.321411783
N_RAN3	.971678688	.013692796	.944840010	.998517367
N_RAN4	.832408021	.041509688	.751046614	.913769428
N_RANX	.933399147	.013394346	.907145447	.959652846
COM_RCK	1.146943198	.021072587	1.105639699	1.188246697
N_RDL12X	1.457057418	.039550042	1.379537030	1.534577806
N_RDLX	1.714531965	.020860249	1.673644661	1.755419268
N_REN23	1.271761220	.020141896	1.232281930	1.311240509
N_REN5	1.103923841	.040405050	1.024727588	1.183120095
N_RENX	1.164442036	.019328996	1.126556078	1.202327995
COM_RIC	1.337711611	.011437614	1.315293221	1.360130001
N_RID3	1.650976512	.102326886	1.450409853	1.851543171
N_RIDX	2.178112719	.037061945	2.105469147	2.250756290
N_RIV5	1.110502059	.023295864	1.064840809	1.156163310
N_RIVX	1.023915823	.007144370	1.009912442	1.037919205
N_RMT2	1.384466726	.092195931	1.203757328	1.565176125
N_RMTX	1.213406672	.027726126	1.159061848	1.267751496
N_ROX3	1.849737219	.072168545	1.708282664	1.991191773
N_ROXX	2.324127320	.045319893	2.235297689	2.412956951
N_ROYX	1.144610904	.015259092	1.114702195	1.174519613
N_RUNX	.919446429	.010083593	.899681998	.939210859
COM_RUT	1.169026938	.024047031	1.121893355	1.216160520
N_SCA56	1.492428673	.057460848	1.379802063	1.605055283
N_SCAX	1.727052862	.031984074	1.664362213	1.789743510
N_SCE12	1.128194973	.014755169	1.099273982	1.157115965
N_SCEX	1.052091464	.007640745	1.037115159	1.067067770
N_SDC569	1.142336465	.012431238	1.117970515	1.166702415
N_SDCX	1.029119338	.006251273	1.016866478	1.041372197
COM_SHG	1.339688255	.024291682	1.292075142	1.387301368
N_SHN135	.982404803	.010143701	.962522558	1.002287048
N_SHNX	.961684914	.009696178	.942679840	.980689987
COM_SHS	1.047493416	.012116438	1.023744490	1.071242341
COM_SIG	1.118807633	.005541264	1.107946432	1.129668834
N_SIL7	1.151177278	.035754253	1.081096858	1.221257698
N_SILX	.998868702	.008567803	.982075308	1.015662095
N_SNA2	1.376116604	.057104046	1.264189346	1.488043861
N_SNAX	1.547634229	.037755988	1.473630292	1.621638166
COM_SOC	1.361772008	.015764323	1.330873015	1.392671000
COM_SOM	.972430419	.005694021	.961269806	.983591033
N_SOV1	.914179880	.025942520	.863331030	.965028731
N_SOVX	.985242301	.028939088	.928520002	1.041964601
N_SOWX	1.059365681	.014227995	1.031477982	1.087253379
N_SPH3	1.505419888	.026873808	1.452745659	1.558094118
N_SPHX	1.138791772	.016167429	1.107102670	1.170480875
COM_SPR	1.202569854	.026938718	1.149768397	1.255371311
N_SSD37	1.472077918	.035290976	1.402905548	1.541250287
N_SSDX	1.585763276	.027355976	1.532143968	1.639382584
COM_STA	1.539726504	.027547173	1.485732439	1.593720570
N_STR1	1.196042559	.013324911	1.169924957	1.222160160
N_STR3	1.074176512	.020444969	1.034103181	1.114249842
N_STRX	1.118603300	.013273780	1.092585917	1.144620682
N_TAR1	.941395075	.018394566	.905340654	.977449495
N_TEMX	.916532254	.008010000	.900832187	.932232320
N_THOX	1.065409585	.012182099	1.041531961	1.089287210
N_TUS14	1.051442895	.009435156	1.032949440	1.069936351
N_TUSX	1.029942018	.006203321	1.017783146	1.042100890
COM_TUX	1.196129660	.014656707	1.167401660	1.224857660

Parameter	Estimate	Asymptotic standard error	Asymptotic 95 % confidence interval Lower	Upper
N_VAL25	1.034019036	.009124610	1.016134268	1.051903804
N_VALX	1.109819796	.011158436	1.087948611	1.131690980
N_VAR23X	1.411993540	.012691175	1.387118097	1.436868982
N_VARX	1.137228837	.010497861	1.116652418	1.157805256
N_VIS3	1.044121234	.058170613	.930103441	1.158139026
N_WGT12X	1.212744971	.052678301	1.109492432	1.315997511
N_WGTX	1.153295617	.018463109	1.117106848	1.189484386
N_WHI4	.914640207	.008871903	.897250761	.932029653
N_WHIX	.945548878	.013811533	.918477468	.972620288
N_WHL46X	1.457646284	.014758316	1.428719124	1.486573444
N_WHL7	1.219899960	.061107926	1.100124864	1.339675056
N_WHLX	1.375624207	.018168223	1.340013431	1.411234982
N_WIL34	1.283649200	.015583416	1.253104798	1.314193603
N_WIL510	.990690446	.028207805	.935401504	1.045979388
N_WILX	1.108236355	.014055579	1.080686601	1.135786108
COM_WIN	1.200371053	.015640125	1.169715497	1.231026609
N_WLD34	1.644074050	.055235870	1.535808526	1.752339574
N_WLD15	1.252857672	.024382876	1.205065814	1.300649529
N_WLDX	1.388912183	.020186248	1.349345961	1.428478405
COM_WND	1.222663488	.021231464	1.181048581	1.264278394
N_WOO1	.984058605	.012727073	.959112801	1.009004410
N_WOOX	1.105885615	.012097218	1.082174362	1.129596867
COM_WSP	1.059331443	.032300724	.996020142	1.122642743
N_UMR2	1.383973993	.060494542	1.265401166	1.502546821
N_UMRX	2.058856294	.013651688	2.032098189	2.085614398
COM_UNI	1.466291969	.020693772	1.425730971	1.506852968